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QUESTIONS AND DISCUSSIONS.

Edited by U. G. Mitchell, University of Kansas.

DISCUSSIONS.

I. RELATING TO THE INTERPRETATION OF THE DEGENERATE CONICS.

By H. W. Brinkmann, Student in the Polytechnic High School, Riverside, Calif.

In textbooks on analytic geometry the degenerate cases which appear on analyzing the general equation of the second degree are usually treated from the algebraic side alone. It is easily shown, however, that all the degenerate cases can be made to fit precisely into the definitions of the proper conics.

Consider first the case of two intersecting lines. These can be interpreted as a hyperbola as follows. If we take the bisector of one angle between them, the ratio of the distance from any point on either line to their point of intersection and the distance to the bisector is constant, viz., the secant of one half the angle between the lines. In other words, the lines can be called an hyperbola whose focus is the intersection, whose directrix is the bisector of the angle between the lines, and whose eccentricity is the secant of one half that angle. Since the secant of an angle can never be less than unity, it is clear that the lines can represent only an hyperbola.

Evidently the bisector of the second angle could be taken as directrix with the same focus. Thus we see that the pair of lines are really two coincident, conjugate hyperbolas.

The second degenerate case, namely a point, is most easily interpreted as an ellipse whose semi-axes are zero. For, if the point is also regarded as the coincident foci of the ellipse, it is clear that the sum of the distances from the point to the foci is constant, namely zero. Therefore the point is an ellipse whose semi-axes are zero.

The only degenerate case that remains is that of two parallel lines. If these are coincident, it evidently represents a parabola whose directrix is any line perpendicular to it and whose focus is their point of intersection, for then the distance from any point on this line to the directrix equals the distance from that point to the focus.

If the lines are non-coincident they are best interpreted as the limiting case of a pair of intersecting lines, if the point of intersection moves indefinitely to one side in the plane. The angle between those lines is then zero and its secant is unity. Hence the hyperbola (intersecting lines) has degenerated into a parabola, for its eccentricity is unity.

It may also be noticed that for the equation of a conic where the focus lies on the directrix we get

$$x^2(1 - e^2) + y^2 = 0$$

where e is the eccentricity. For, taking the directrix as the y-axis and the focus as the origin, we have $\sqrt{x^2 + y^2}$ as the distance from any point (x, y) to the focus and x as the distance from that point to the directrix. Hence, if the point is on the locus,

$$\sqrt{x^2 + y^2} = ex$$

and squaring we get the required equation.

For $e > 1$, $e = 1$, $e < 1$, this reduces to the equations of two intersecting lines, two coincident lines and a point respectively.

II. TWO USEFUL RELATIONS IN TRIGONOMETRY.

By Albert Babbitt, University of Minnesota.

If ABC be any triangle, and if θ be the angle which a straight line CP (Fig. 1) drawn from the vertex C makes with the base of the triangle, and if we let $\angle PCA = \alpha$, $\angle BCP = \beta$, $AP = m$, $PB = n$, we then have

$$(m + n) \cot \theta = m \cot \alpha - n \cot \beta.$$

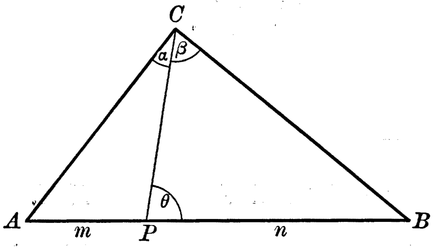

FIG. 1.

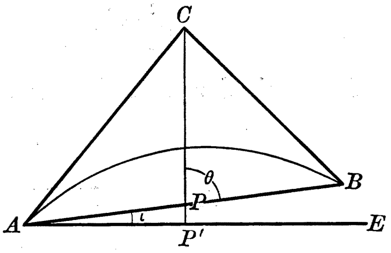

FIG. 2.

The proof of this trigonometric relation is very simple.[1] From $\triangle ACP$, we have

$$\frac{CP}{m} = \frac{\sin A}{\sin \alpha},$$

hence,

$$CP = \frac{m \sin A}{\sin \alpha} = \frac{m \sin (\theta - \alpha)}{\sin \alpha} = m (\sin \theta \cot \alpha - \cos \theta). \qquad (1)$$

Similarly, from $\triangle PCB$, we have

$$CP = \frac{n \sin B}{\sin \beta} = \frac{n \sin (\theta + \beta)}{\sin \beta} = n (\sin \theta \cot \beta + \cos \theta). \qquad (2)$$

From (1) and (2) it follows that

$$m(\sin \theta \cot \alpha - \cos \theta) = n(\sin \theta \cot \beta + \cos \theta).$$

Simplifying and dividing through by $\sin \theta$ (since $\sin \theta \neq 0$), we get

$$(m + n) \cot \theta = m \cot \alpha - n \cot \beta.$$

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[1] Other proofs are given in, for instance, Loney's *Elements of Statics and Dynamics*, tenth edition (1896), page 92. EDITOR.